

# CSE 2500: Problem Set Three

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## Exercise 1

Let  $p$  be a permutation, and let  $p^k$  be the composition of  $p$   $k$  times with itself. Recall that the *order* of the permutation  $p$  is the smallest natural number  $k \geq 1$  such that  $p^k = \text{id}$ , where  $\text{id}$  denotes the identity permutation (mapping each element onto itself).

- (a) Determine the order of the permutation  $(2\ 3\ 1\ 5\ 4\ 7\ 8\ 9\ 6)$  (i.e., the permutation that maps 1 to 2, 2 to 3, 3 to 1 etc).
- (b) Construct a permutation that has a cycle decomposition consisting of cycles of length  $16, 8, 4, 2, 1$ . What is the order of this permutation? Generalize your construction so that it has cycles of length  $2^n, 2^{n-1}, \dots, 2^1, 1$  where  $n$  is any natural number.

## Exercise 2

Find a bijection between the set of all permutations over  $n$  elements and the set  $[n] \times [n-1] \times \dots \times [1]$  where  $[a] = \{1, \dots, a\}$  for a natural number  $a$ . Recall that finding a bijection requires defining the function and then proving that it is 1-1 and onto.