CSE 2500: Problem Set Ten

Instructor: Aggelos Kiayias

Due: Nov. 20

Exercise 1

Suppose a 5-card *poker* hand is drawn at random from a deck of 52 cards. Compute the probability of the following events:

(a) Four of a kind—four cards of the same kind, e.g. A♣, A♠, A♢, A♡, Q♡.

(b) Three of a kind—e.g., A♣, A♠, A♢, K♡, Q♡.

(c) Two pairs—two pairs of cards of the same kind, e.g. A♣, A♠, K♢, K♡, Q♡.

(d) One pair— e.g., A♣, A♠, K♢, Q♡, J♡.

(e) A straight— five cards in sequence, where suit is unimportant, e.g., A♣, 2♢, 3♠, 4♡, 5♣. Note that A can be a high or a low card.

Exercise 2

Consider the following (finite) probability space: $(\mathbb{G}_n, p)$ where $\mathbb{G}_n$ contains *all* graphs of n vertices (without edge repeats or self-loops) and for each $G \in \mathbb{G}_n$, it holds that $p_G = 2^{-\binom{n}{2}}$. A random graph according to this finite probability space can be sampled by implementing a con tossing device as follows: for each pair of vertices $\{v_i, v_j\}$ flip a coin, and if the result is heads draw the edge.

Given a graph $G = (V, E)$ over n vertices $\{1, \ldots, n\}$ a certain vertex i is said to have the vertex j as a superior if $i < j$ and $\{i, j\} \in E$. A graph G is called modest if for any $i \in \{1, \ldots, n\}$ no more than 50% of the vertices $\{i + 1, \ldots, n\}$ are connected to i (i.e., the number of superiors of i is no more than 50%). Compute the probability that a random graph is modest: give any precise formula that describes this probably and also find a reasonably tight simple lower bound. Also answer the question what is the exact probability that a random graph of $n = 5$ vertices is modest.