CSE 2500: Problem Set Three

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Exercise 1

Let R and S be arbitrary equivalences on a set X. Decide which of the following relations are necessarily also equivalences: (a) $R \cap S$, (b) $R \cup S$, (c) $R \setminus S$, and (d) $R \circ S$. If yes, prove it; if not, give a counterexample.

Exercise 2

Call an equivalence $\asymp$ on the set $\mathbb{Z}$ (the integers) a *congruence* if the following condition holds for all $a, x, y \in \mathbb{Z}$: if $x \asymp y$ then also $a + x \asymp a + y$.

(a) Let q be a nonzero integer. Define a relation $\equiv_q$ on $\mathbb{Z}$ by letting $x \equiv_q y$ if and only if q divides $x - y$. Check that $\equiv_q$ is a congruence according to the above definition.

(b) Suppose we replaced the condition "$a + x \asymp a + y$" in the definition of a congruence by "$an \asymp ay$". Would the claim in (a) remain true for this kind of "multiplicative congruence"?

Note that $\mathbb{Z}$ is a set of all negative integers, positive integers, and a number 0.