**Fall 2008 — A. Kiayias and L. Michel.**

## Joint Extra Credit Homework for CSE 2100 - CSE 2500

A general "subtraction game" is specified by a number $m \in \mathbb{N}$ and a finite set $S \subseteq \mathbb{N}$ where $m$ is called the initial game configuration and $S$ constitutes the set of allowed moves. It is assumed that $1 \in S$. A subtraction game $\langle m, S \rangle$ is a two player game with the following rules:

- Initial setup: A pile of $m$ coins is put on a table.
- Player 1 is the player that makes the first move.
- At every move, a player removes a number $k$ of coins from the table so that $k \in S$.
- The winner is the player that removes the last coin from the pile.

**Task**. Any subtraction game has a winning strategy for one of the two players. Write a program that takes as input the specification $\langle m, S \rangle$ and returns the player that has a winning strategy (i.e., your program should return either "player 1" or "player 2").

Extra Extra Credit: write an interactive program that allows a human to play with the computer a subtraction game. The program should offer to the user to select $m$ and $S$ as well as specify who is player 1 and player 2 (computer or human). If the computer has a winning strategy your program should always win over the user. If the user has a winning strategy, your program should always win if the user makes one wrong move.

In order to solve this problem you should consider the following:

- For each game $\langle m, S \rangle$ there is a total of $m + 1$ game configurations corresponding to the number of coins left in the pile, i.e., elements of the set $\{0, \ldots, m\}$.
- The game configuration graph of each game $\langle m, S \rangle$ is a directed graph with $m+1$ vertices $V = \{0, \ldots, m\}$ and a set of edges $E$ such that $(i, j) \in E$ if and only if $i - j \in S$.
- Vertices can be labeled as P-positions or N-positions. A P-position means that the player that moved previously has a winning strategy or won the game, whereas an N-position means that the player that moves next has a winning strategy. Clearly the vertex 0 is a P-position. Labeling of other vertices in the graph is possible using the following two rules:
  1. A vertext is a P-position if all vertices that it can reach in one step are N-positions.
  2. A vertex is an N-position if it can reach a vertex that is a P-position in one step.
- Player 1 has a winning strategy if and only if vertex $m$ is labeled as N-position.