CSE 2500 — Kiayias.

Pre-Midterm Review Exercises. Use them (as well as all previous homeworks) as your study guide. All the exercises will also be solved in class.

Exercises 11 and 12 must be handed in as homework on October 9.

1. Let $F(0) = 0, F(1) = 1, F(2) = 1, F(3) = 2, \ldots$ be the Fibonacci sequence. Prove that for all $n \geq 1$ it holds that $F(3n)$ is an even number.

2. Recall that functions $X \to Y$ can be seen as sets of pairs, i.e., subsets of $X \times Y$. If $f, g \subseteq X \times Y$ are functions is it true that $f \cup g$ and $f \cap g$ are also functions?

3. Suppose you begin with a file of n stones and split this pile into n piles of one stone each by successively splitting a pile of stones into two smaller piles. Each time you split a pile you multiply the number of stones in each of the two smaller piles you form, so that if these piles have r and s stones in them, respectively, you compute $r \cdot s$. Show that no matter how you split the piles, the sum of the products computed at each step equals $n(n-1)/2$.

4. (a) If $g \circ f$ is an onto function, does g have to be onto? Does f have to be onto? (b) If $g \circ f$ is an one-to-one function, does g have to be one-to-one? Does f have to be one-to-one?

5. For a relation R on a set X we define the symbol R^n as follows: $R^1 = R$ and $R^{n+1} = R \circ R^n$ (see definition 6 of Chapter 8.1 in the text).

   (a) Prove that if X is finite and R is a relation on it, then there exist $r, s \in \mathbb{N}$ $r < s$ such that $R^r = R^s$.

   (b) Find a relation R on a finite set such that $R^n \neq R^{n+1}$ for every $n \in \mathbb{N}$.

6. Formulate the conditions for reflexity of a relation, for symmetry of a relation, and for its transitivity using the adjacency matrix of the relation.

7. Call an equivalence $\backsim$ on the set $\mathbb{Z}$(the integers) a *congruence* if the following condition holds for all $a, x, y \in \mathbb{Z}$: if $x \backsim y$ then also $a + x \backsim a + y$. Let q be a nonzero integer. Define a relation $\equiv_q$ on $\mathbb{Z}$ by letting $x \equiv_q y$ if and only if q divides $x - y$. Check that $\equiv_q$ is a congruence according to the above definition. What are the equivalence classes of the congruence $\equiv_q$?

8. Cars are compared according to two properties: gas consumption per 100 miles m and acceleration a. A car is represented by two real numbers $\langle m, a \rangle$. A car $\langle m, a \rangle$ is at least as good as a car $\langle m', a' \rangle$ if $m \leq m'$ and $a \geq a'$; in this case we write that $\langle m, a \rangle \succeq \langle m, a' \rangle$. Prove that $\succeq$ is a partial order.

9. Determine the number of ordered pairs (A, B), where $A \subseteq B \subseteq \{1, 2, \ldots n\}$.

10. We have k balls, and we distribute them into n (numbered) bins. Fill out the formulas for the number of ditributions for various variants of the problem in the following table:

	At most 1 ball into each bin	Any number of balls into each bin
Balls are distinguishable (have distinct colors)		
Balls are indistinguishable		

11. Use the Inclusion / Exclusion principle to count the number of all derangements over $\{1, 2, 3, 4\}$. A derangement over n elements is a permutation for which it holds that $\pi(i) \neq i$ for all $i \in \{1, \ldots, n\}$.

12. Suppose that $p_1, p_2, \ldots, p_k$ are distinct primes. Use the principle of inclusion-exclusion to find $\phi(p_1 \ldots p_k)$, the number of positive integers not exceeding $p_1 \ldots p_k$ that are relatively prime to $p_1 \ldots p_k$.