Pre-Midterm Review Exercises.

In addition to the exercises you solved so far, we will also discuss the following exercises in class.

1. Let $F(0) = 0, F(1) = 1, F(2) = 1, F(3) = 2, \ldots$ be the Fibonacci sequence. Prove that for all $n \geq 1$ it holds that $F(3n)$ is an even number.

2. Recall that functions $X \rightarrow Y$ can be seen as sets of pairs, i.e., subsets of $X \times Y$. If $f, g \subseteq X \times Y$ are functions is it true that $f \cup g$ and $f \cap g$ are also functions?

3. Exercise 5 from page 31.

4. Exercise 3a-3b from page 36.

5. Exercise 1 from page 39.

6. What are the equivalence classes of the congruence $\equiv_q$ in page 40?

7. Cars are compared according to two properties: gas consumption per 100 miles $m$ and acceleration $a$. A car is represented by two real numbers $\langle m, a \rangle$. A car $\langle m, a \rangle$ is at least as good as a car $\langle m', a' \rangle$ if $m \leq m'$ and $a \geq a'$; in this case we write that $\langle m, a \rangle \geq \langle m', a' \rangle$. Prove that $\geq$ is a partial order.

8. Exercise 2 on page 52.

9. Exercises 17 and 18 pages 63-64.

10. Use the Inclusion / Exclusion principle to count the number of all derangements over $\{1, 2, 3, 4\}$. A derangement over $\{1, \ldots, n\}$ is a permutation for which it holds that $\pi(i) \neq i$ for all $i \in \{1, \ldots, n\}$.

11. Find the disjunctive normal form of the formula $\phi = (\neg x_1 \lor x_2) \rightarrow x_1 \lor x_2$ (use the truth-table).

12. Write this statement “All mathematicians are crazy” as a formula of predicate logic using the predicates $M(x) = "x$ is a mathematician" and $C(x) = "x$ is crazy." Negate the formula. Provide the negation both as a logical formula as well as an English statement.

13. The predicate $\text{Love}(x, y, t)$ stands for “$x$ loves $y$ at time $t$.” Using this predicate formalize the statements (i) “Everybody loves somebody sometime” (ii) “Nobody is in love all the time” (iii) “At some point everybody falls in love with someone forever” (iv) “some people never fall in love”

note: You may use the symbol $\geq$ if you need it.