A puzzle kind description

Puzzle: is a collection of dominos like:

$\{[\frac{b}{ca}], [\frac{a}{ab}], [\frac{ca}{a}], [\frac{abc}{c}]\}$

A match is a list of these dominos (repetitions permitted) so that the string we get by reading off the symbols on the top is the same as the string of symbols on the bottom

Example: $\{[\frac{a}{ab}][\frac{b}{ca}][\frac{ca}{a}][\frac{a}{ab}][\frac{abc}{c}]\}$

Reading off the top string we get: `abcaaabc`

Reading off the bottom string we get: `abcaaabc`

which is the same.

Note

- We can also depict a match by deforming the dominos so that the corresponding symbols from top and bottom line up, Figure 1

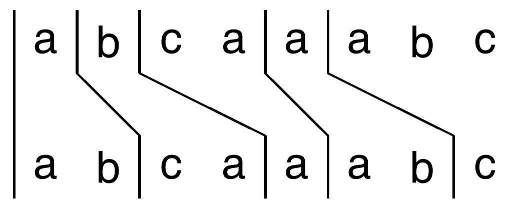

Figure 1: Deforming dominos

Observation

For some puzzle finding a match may not be possible. For example, the puzzle

$\{[\frac{abc}{ab}], [\frac{ca}{a}], [\frac{acc}{ba}]\}$

cannot contain a match because every top string is longer than the corresponding bottom string

Post correspondence problem:

Determine whether a collection of dominos has a match.

Note: this problem is unsolvable by algorithms

Mathematical formulation

An instance of PCP is a collection of dominos:
$P = \{[\frac{t_1}{b_1}], [\frac{t_2}{b_2}], \ldots, [\frac{t_k}{b_k}]\}$
A match is a sequence $i_1, i_2, \ldots, i_l$ of P components where
$t_{i_1} t_{i_2} \ldots t_{i_l} = b_{i_1} b_{i_2} \ldots b_{i_l}$

PCP determine whether P has a match.

Notation:

$$PCP = \{\langle P \rangle | P\ instance\ of\ PCP\ with\ a\ match\}$$

Theorem 5.11

PCP is undecidable

Proof idea: by reduction from A_{TM} via accepting computation histories

- Show that from a TM M and input w we can construct an instance P of PCP where a match is an accepting computation history for M on w
- If we could determine whether this instance of PCP has a match, we would be able to determine whether M accepts w
- Since A_{TM} is undecidable we cannot determine whether P has a match

Constructing P

- Choose dominos in P so that making a match forces a simulation of M to occur
- In the match, each domino links a position or positions in one configuration with the corresponding one(s) in the next configuration