

Number Representation

Three systems to represent numbers in computers:

(1) **Sign-and-magnitude**

(2) **1's-complement**

(3) **2's-complement**

- In all three systems, the most significant bit (MSB) is 0 for positive numbers and 1 for negative numbers.

- Positive values have identical representation in all three systems.

- In the sign-and-magnitude system, negative values are represented by changing the MSB from 0 to 1.

- In the 1's-complement system, negative values are obtained by complementing each bit of the corresponding positive number. (subtracting the positive value from $2^n - 1$, n is # of bits)

- In the 2's-complement system, negative values are obtained by subtracting the corresponding positive values from 2^n . (or by adding 1 to the 1's-complement of that number)

Number Representation (Cont'd)

- There are distinct representations for +0 and -0 in both the sign-and-magnitude and 1's-complement systems.
- The 2's-complement system has only one representation for 0.
- For n -bit numbers, the value -2^{n-1} is representable in the 2's-complement system but not in the other systems.
- The sign-and-magnitude system is the most natural; the 1's-complement system is easily related to the natural system; the 2's-complement most facilitates the addition and subtraction operations.
- Nowadays, **most computers use 2's-complement system.**

Addition and Subtraction of Signed Numbers

The *addition* and *subtraction* rules of n-bit signed numbers using the 2's-complement representation system:

- (1) To *add* two numbers, add their n-bit representations, ignoring the carry-out signal from the MSB position. The sum will be the algebraically correct value in the 2's-complement representation as long as the answer is in the range -2^{n-1} through $+2^{n-1} - 1$.
- (2) To *subtract* two numbers X and Y , i.e., to perform $X - Y$, form the 2's-complement of Y and then add it to X , as in (1). Again, the result will be the algebraically correct value in the 2's-complement representation system if the answer is in the range -2^{n-1} through $+2^{n-1} - 1$.

Addition and Subtraction of Signed Numbers (Cont'd)

In some cases, we need to represent a number in the 2's-complement system by using a number of bits larger than some given size. For a positive number, this is done by adding leading 0's. For a negative number, we replicate the sign bit (1, namely) to the left as many times as desired. (Why is this correct?)

The simplicity of adding and subtracting signed numbers in 2's-complement representation is the reason why this number representation is popular. (Why is this correct?)

Arithmetic Overflow

In the 2's-complement number representation system, n bits can represent values in the range -2^{n-1} to $+2^{n-1} - 1$. For example, using four bits, the range of numbers that can be represented is -8 to +7. When the result of an arithmetic operation is outside the representable range, an *arithmetic overflow* has occurred.

A simple way to detect overflow is to examine the signs of the two summands X and Y and the sign of the result S . When both operands X and Y have the same sign, an overflow occurs when the sign of S is not the same as the signs of X and Y .

(Note: *Overflow can occur only when adding two numbers that have the same sign.*)