Name: ______________________________________

CSE 3500 Algorithms and Complexity
Exam III, December 15, 2009

Note: You are required to give proofs to the time and processor bounds of your algorithms. Read
the questions carefully before attempting to solve them.

1. (17 points) The SubsetSum problem takes as input a set $X = \{k_1, k_2, \ldots, k_n\}$ of integers
   and another integer $K$. The problem is to check if there exists a subset $X'$ of $X$ whose
   elements sum to $K$. For example, if $X = \{5, 3, 11, 8, 2\}$ and $K = 16$ then the answer is YES
   since the subset $X' = \{5, 11\}$ has a sum of 16. Present a dynamic programming algorithm
   for SubsetSum whose run time is $O(nK)$.
2. (16 points) Input is an undirected graph $G(V, E)$ in which each edge has the same weight $w$ where $w$ is a positive real number. Present an $O(|V| + |E|)$ time algorithm to solve the single-source shortest path problem on $G$ with $s$ as the source node.
3. (17 points) Present a common-CRCW PRAM algorithm to compute the product of two
$n \times n$ Boolean matrices in $O(1)$ time. You can use up to $n^3$ processors. Note that the
product will also be a Boolean matrix.
4. (17 points) Input is a sequence $X$ of keys $k_1, k_2, \ldots, k_n$. It is given that $X$ has only $O(1)$ distinct keys. Present a PRAM algorithm to sort $X$ that runs in time $O(\log n)$ using $\frac{n}{\log n}$ processors. What version of the PRAM are you using?
5. (16 points) Define DSAT as the following problem:

Input is a Boolean formula $F$ on $n$ variables that is in disjunctive normal form (DNF). The problem is to decide if $F$ is satisfiable.

Is DSAT in $\mathcal{P}$? If so, present a polynomial time algorithm for its solution. If not, show that it is $\mathcal{NP}$-complete.
6. (17 points) Let $G(V, E)$ be any undirected graph. $I$ is said to be an independent set of $G$ if $I \subseteq V$ and no two nodes in $I$ are connected by an edge. Let INDSET be the following problem:

Input are an undirected graph $G(V, E)$ and an integer $k, 1 \leq k \leq |V|$. The problem is to decide if $G$ has an independent set of size $k$.

Prove that CLIQUE $\propto$ INDSET.