1. Input is a (not necessarily sorted) sequence $S = k_1, k_2, \ldots, k_n$ of $n$ arbitrary numbers. Consider the collection $C$ of $n^2$ numbers of the form $\min\{k_i, k_j\}$, for $1 \leq i, j \leq n$. Present an $O(n)$-time and $O(n)$-space algorithm to find the median of $C$.

2. Two sets $A$ and $B$ have $n$ elements each. Assume that each element is an integer in the range $[0, n^{100}]$. These sets are not necessarily sorted. Show how to check whether these two sets are disjoint in $O(n)$ time. Your algorithm should use $O(n)$ space.

3. Let $\hat{F}(I)$ be the value of the solution generated on problem instance $I$ by GreedyKnapsack when the objects are input in nonincreasing order of the $p_i$'s. Let $F^*(I)$ be the value of an optimal solution for this instance. How large can the ratio $F^*(I) / \hat{F}(I)$ get?

4. Find a minimum spanning tree for the following graph $G(V, E)$ either using Prim’s algorithm or using Kruskal’s algorithm: $V = \{1, 2, 3, 4, 5\}$. The edge weights are: $W(1, 2) = 11; W(1, 4) = 2; W(1, 3) = 2; W(2, 3) = 5; W(3, 4) = 4; W(3, 5) = 5; W(4, 5) = 7$.

5. Use Dijkstra’s algorithm to solve the single source shortest path problem on the directed graph $G(V, E)$: $V = \{s, 1, 2, 3, 4, 5\}$. Edge weights are: $W(s, 1) = 2; W(s, 2) = 15; W(1, 3) = 6; W(1, 4) = 3; W(2, 4) = 4; W(2, 5) = 2; W(3, 4) = 2; W(4, 2) = 5; W(4, 3) = 1; W(4, 5) = 5.$