1. What is the run time of the following algorithm?

**Algorithm** Sum();

Result := 0;

for $i := 1$ to $n$ do
    for $j := 1$ to $i$ do
        for $k := 1$ to $j$ do
            Result++;
    
Output Result;

2. Show that:
   
   (a) $14n^3\log n + 5n^2 = \Theta(n^3\log n)$;
   
   (b) $\log(n!) = \Theta(n \log n)$;
   
   (c) $(\sqrt{n})^{\sqrt{n}} = o\left(2^{n^{0.6}}\right)$.

3. Prove or disprove:

   a) If $f(n) = O(g(n))$ then $2f(n) = O(2g(n))$.

   b) $\max\{f(n), g(n)\} = \Theta(f(n) + g(n))$.

   c) For any real constants $a$ and $b$, $(n + a)^b = \Theta(n^b)$.

4. Input are an array $a[1 : n]$ of arbitrary real numbers and another real number $u$. The problem is to check if the array has two elements $x$ and $y$ such that $x + y = u$. Present an algorithm to solve this problem. What is the run time of your algorithm?

5. Input is an array $a[1 : n]$ of arbitrary real numbers. It is given that the array has two elements such that each of these elements is repeated $\frac{n}{4}$ times. The other elements are unique. (In other words the array has $\frac{n}{4} + 2$ distinct elements). Present a Las Vegas algorithm to identify the two repeated elements. The run time of your algorithm should be $\tilde{O}(\log n)$. Prove the run time of your algorithm.