1. Show how to form a heap out of the elements 4, 11, 3, 5, 8, 23, 7, 9, 14, 15 using the Heapify algorithm discussed in class. Show the individual steps.

2. Find an efficient data structure for representing a subset $S$ of the integers from 1 to $n$. Operations we wish to perform on the set are
   - INSERT($i$) to insert the integer $i$ to the set $S$. If $i$ is already in the set, this instruction must be ignored.
   - DELETE to delete an arbitrary member from the set.
   - MEMBER($i$) to check whether $i$ is a member of the set.

   Your data structure should enable each one of the above operations in constant time (irrespective of the cardinality of $S$).

3. Input is a sequence $X$ of $n$ keys with many duplications such that the number of distinct keys is $d (< n)$. Present an $O(n \log d)$-time sorting algorithm for this input. (For example, if $X = 5, 6, 1, 18, 6, 4, 4, 1, 5, 17$, the number of distinct keys in $X$ is six.)

4. Solve the following recurrence relations:
   
   (a) \[ T(n) = \begin{cases} 
   1 & n \leq 4 \\
   12T(n/4) + n^{1.5} & n > 4 
   \end{cases} \]
   
   (b) \[ T(n) = \begin{cases} 
   1 & n \leq 4 \\
   T(\sqrt{n}) + \log n & n > 4 
   \end{cases} \]

5. Given two sets $A$ and $B$ with $m$ and $n$ elements (respectively) from a linear order. These sets are not necessarily sorted. Also given that $m \leq n$. Show how to compute $A \cup B$ and $A \cap B$ in $O(n \log m)$ time.

6. $X_1, X_2, \ldots, X_\ell$ are sorted sequences such that $\sum_{i=1}^\ell |X_i| = n$. Show how to merge these $\ell$ sequences in time $O(n \log \ell)$.