1. [Problem 8.17 from MR95]. In defining a random leveling for a skip list, we sampled the elements from $L_i$ with probability $1/2$ to determine the next level $L_{i+1}$. Consider instead the skip list obtained by performing the sampling with probability $p$ (at each level), where $0 < p < 1$. (a) Determine the expectation of the number of levels $r$, and prove a high probability bound on $r$; (b) Determine as precisely as you can the expected cost of each operation in this skip list; and (c) Discuss the relationship between the choice of $p$ and the performance of the skip list in practice.

2. [Exercise 8.13 from MR95]. Assume for simplicity that $n = s$. Show that for $m = 2^{\Omega(s)}$, there exist perfect hash families of size polynomial in $m$. (Hint: Use the probabilistic method.)

3. [Problem 8.22 from MR95]. In this problem we consider a weakening of the notion of 2-universal families of hash functions. Let $g(x) = x \mod p$. For each $a \in Z_p$, define the function $f_a(x) = ax \mod p$, and $h_a(x) = g(f_a(x))$, and let $H = \{h_a|a \in Z_p, a \neq 0\}$. Show that $H$ is nearly-2-universal in that, for all $x \neq y$, $\delta(x,y,H) \leq \frac{2|H|}{n}$.

4. The chain sorting problem is defined as follows: The input is a sequence $X$ of $n$ arbitrary elements and the output is the right neighbor of each element of $X$ in sorted order. For example, if $X = 5, 11, 4, 3, 23, 17, 8, 45, 14$, then, the output is $8, 14, 5, 4, 45, 23, 11, \infty, 17$. Show how to solve this problem in $\tilde{O}(1)$ time using $n^2$ arbitrary CRCW PRAM processors.

5. Input is a sequence $X$ of $n$ keys where each key is an integer in the range $[1, n^c]$, $c$ being any constant. Show how to sort $X$ in $O(\sqrt{n})$ time using $\sqrt{n}$ CREW PRAM processors.

6. Input are two sets $A$ and $B$ with $|A| = n$, $|B| = m$, and $m < n$. These two sets contain arbitrary real numbers and are not necessarily in sorted order. Present an $\tilde{O}(\log m)$ time algorithm to compute $A \cap B$. You can use up to $n$ arbitrary CRCW PRAM processors. As an example, if $A = \{8, 12, 3, 6, 11, 15, 4, 55, 32, 18\}$ and $B = \{11, 18, 5, 15, 7, 3\}$, then the elements 18, 11, 3, and 15 should be output (in any order) in successive cells of the common memory.